AN ADAPTIVE IMPEDANCE CONTROL SCHEME FOR CONSTRAINED ROBOTS

L. Huang[a], S. S. Ge[b] and T. H. Lee[b]

[a]School of Electrical and Electronic Engineering, Singapore Polytechnic, Singapore 139651
loulin@sp.edu.sg

[b]Dpartment of Electrical and Computer Engineering, National University of Singapore Singapore 117576
elegesz@nus.edu.sg, eleleeth@nus.edu.sg

ABSTRACT
This paper addresses the issue of position/force tracking for a constrained robot within the framework of impedance control. An adaptive controller is developed in which a varying desired impedance is adaptively tuned with the robot's position tracking error. It guarantees the asymptotic convergence of the robot position tracking error and the boundedness of the constraint force tracking error. Simulation results are provided to verify the effectiveness of the scheme.

Keywords: Adaptive control, Impedance control, Constrained robot.

1. INTRODUCTION

Impedance control is one of the effective control approaches for position/force control of constrained robots. It is aimed to achieve a desired generalized dynamic impedance without directly controlling the robot's position and constraint force. Model-based computed torque control [1,2,3] was the main control method. To handle uncertainties, various adaptive or robust impedance control schemes were developed in the literature [4,5]. Some new control methods such as neural network control have also been introduced into impedance control schemes [6].

Impedance control is applicable in both constrained and unconstrained motion of robots and shows good robustness to uncertainties and disturbances [7]. Traditional impedance control cannot guarantee the tracking of the robot's position and the constraint force. Some approaches have been proposed to overcome this drawback. In [8], direct control of the force in impedance control was realized by varying the robot's desired trajectories through a PI adaptive control law. In [9], the desired impedance was treated as the dynamic model of a plant driven by the constraint force. The control torque was then derived to directly control the robot's position through a model reference adaptive control. In [3], a parallel control scheme was presented which incorporates hybrid position/force control and impedance control. It requires an on-line modeling of the constraint surface .

In this paper, an adaptive impedance control scheme is developed to achieve the asymptotic convergence of the robot position tracking error and the boundedness of constraint force error. The rest of the paper is organized as follows. In Section 2, the dynamic model of the constrained robot is given. In Section 3, the impedance model of the robot and the environment is derived. In Section 4, the adaptive impedance controller is presented. In Section 4, simulation studies are carried out to show the effectiveness of the proposed controller, followed by the conclusion about the work presented in the paper.

2. DYNAMIC MODEL

Consider the dynamic model of a constrained robotic manipulator [3][11][12]:

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau + J^T(q)f \qquad (1)$$

where $q \in R^n$ are the joint displacements, $\dot{q} \in R^n$ are the joint velocities, $M(q) \in R^{n\times n}$ is the inertia matrix, $C(q,\dot{q}) \in R^{n\times n}$ is the coriolis and centrifugal force matrix, $G(q) \in R^n$ is the gravitational force, $\tau \in R^n$ are the joint torques, $J(q) \in R^{m\times n}$ is the Jacobian matrix, $f \in R^m$ is the constraint force, n is the degree of freedom of the robot, and m is the dimension of the work space.

Through the following kinematic relationships

$$r = \Phi(q), \qquad \dot{r} = J(q)\dot{q} \qquad (2)$$

where $r \in R^m$ and $\dot{r} \in R^m$ are the position and the velocity of the robot's end effector, the robot dynamics in the operational space is derived :

$$M_r(q)\ddot{r} + C_r(q,\dot{q})\dot{q} + G_r(q) = J^{-T}(q)\tau + f \qquad (3)$$

where

$$M_r(q) = J^{-T}(q)M(q)J^{-1}(q)$$
$$C_r(q,\dot{q}) = J^{-T}(q)(M(q)\dot{J}^{-1}(q) + C(q,\dot{q})J^{-1}(q))$$
$$G_r(q) = J^{-T}(q)G(q)$$

The above dynamic model has the following properties [6][11]:

Property 1. The inertia matrix $M_r(q)$ is symmetric and positive definite.
Property 2. The matrix $\dot{M}_r(q) - 2C_r(q,\dot{q})$ is skew-symmetric given that $C(q,\dot{q})$ is defined with Christoffel symbols [3][11].

3. IMPEDANCE MODEL

In traditional impedance control, the desired impedance is chosen as [3]:

$$f_d - f = M_m(\ddot{r}_d - \ddot{r}) + D_m(\dot{r}_d - \dot{r}) + K_m(r_d - r) \qquad (4)$$

where M_m, D_m and K_m are the constant inertia, damping and the stiffness matrices respectively, f_d is the desired constraint force and f is the actual constraint force

The relationship between the constraint force and the positions of the constraint and the robot is modelled as

$$f = K_e(r_e - r) \qquad (5)$$

where K_e is the stiffness matrix and r_e is the rest location of the constraint, and r is the position of the contact point made by the end effector of the robot.

From equations (4) and (5), we have

$$\ddot{e} + A\dot{e} + Be = c \tag{6}$$

where $e = r_d - r$, $A = M_m^{-1} D_m$, $B = M_m^{-1}(K_m + K_e)$ and $c = M_m^{-1}(f_d + K_e(r_d - r_e))$.

4. ADAPTIVE IMPEDANCE CONTROL

The objective of the controller is to make r converge to its desired trajectory r_d and $f_d - f$ be bounded.

Consider the following controller

$$\tau = J^T (M_r u_0 + C_r \dot{r} + G_r - f)$$

where

$$u_0 = \ddot{r}_d + \hat{A}(\dot{r}_d - \dot{r}) + \hat{B}(r_d - r) - \hat{c}$$

with $\hat{A}$, $\hat{B}$ and $\hat{c}$ being the estimates of uncertain parameters A, B and c respectively.

Substituting τ into the dynamic model (3), we have

$$\ddot{e} + \hat{A}\dot{e} + \hat{B}e = \hat{c} \tag{7}$$

Suppose that the reference model of the position tracking error e_m is specified by

$$\ddot{e}_m + A_m \dot{e}_m + B_m e_m = 0 \tag{8}$$

where A_m and B_m are positive definite. Obviously $e_m \to 0$ and $\dot{e}_m \to 0$ when $t \to \infty$.

Subtracting equation (8) from equation (7), we have

$$\ddot{\xi} + A_m \dot{\xi} + B_m \xi = (A_m - \hat{A})\dot{e} + (B_m - \hat{B})e + \hat{c} \tag{9}$$

where $\xi = e - e_m$.

Defining $x = [\xi^T \dot{\xi}^T]^T$ and $y = [e^T \ \dot{e}^T]^T$, equation (9) is re-written in a state space form:

$$\dot{x} = M_x x + M_y y + [0 \ \hat{c}^T]^T \tag{10}$$

where $M_x = \begin{bmatrix} 0^{n\times n} & I^{n\times n} \\ -B_m & -A_m \end{bmatrix}$ and $M_y = \begin{bmatrix} 0^{n\times n} & 0^{n\times n} \\ B_m - \hat{B} & A_m - \hat{A} \end{bmatrix}$.

For the convergence of e, we have the following theorem.

Theorem 1 *For the closed-loop dynamic system* (10), $e \to 0$ *and* $\dot{e} \to 0$ *when* $t \to \infty$ *if the parameters are updated by*

$$\hat{c} = \hat{c}(0) - Q_c(\int_0^t \varpi(\tau)d\tau + \varpi)$$
$$\hat{A} = \hat{A}(0) + Q_a(\int_0^t \varpi(\tau)\dot{e}^T(\tau)d\tau + \varpi\dot{e}^T)$$
$$\hat{B} = \hat{B}(0) + Q_b(\int_0^t \varpi(\tau)e^T(\tau)d\tau + \varpi e^T)$$

where Q_c, Q_a, Q_b *are all positive definite matrices,* ϖ *is a vector defined by*

$$\varpi = P_1^T\xi + P_2^T\dot{\xi}$$

and the terms P_1 *and* P_2 *are the sub-matrices of a symmetric positive matrix*

$$P = \begin{bmatrix} P_0 & P_1 \\ P_1 & P_2 \end{bmatrix}$$

which satisfies the following Lyapunov equation

$$PM_x + M_x^T P = -Q \tag{11}$$

with Q *being a positive definite matrix.*

Proof:

Let $\hat{a}_i$, $\hat{b}_i$, a_{mi} and b_{mi} ($i = 1,2\cdots n$) be the column vectors of matrices $\hat{A}$, $\hat{B}$, A_m and B_m respectively.

Choose the following Lyapunov function candidate

$$V = x^T Px + (c - \hat{c}^*)^T Q_c^{-1}(c - \hat{c}^*) + \sum_{i=1}^n (\hat{a}_i - a_{mi} - \hat{a}_i^*)^T Q_a^{-1}(\hat{a}_i - a_{mi} - \hat{a}_i^*)$$
$$+ \sum_{i=1}^n (\hat{b}_i - b_{mi} - \hat{b}_i^*)^T Q_b^{-1}(\hat{b}_i - b_{mi} - \hat{b}_i^*)$$

where Q_a, Q_b and Q_c are positive definite matrices, and $\hat{c}^*$, $\hat{a}_i^*$ and $\hat{b}_i^*$ are the vectors to be decided later.

Differentiating V with respect to time t and considering equation (11), we have

$$\begin{aligned}\dot{V} &= -x^T Q x + 2\varpi^T((B_m - \hat{B})e + (A_m - \hat{A})\dot{e}) + 2\sum_{i=1}^n (\hat{a}_i - a_{mi} - \hat{a}_i^*)^T Q_a^{-1}(\dot{\hat{a}}_i - \dot{\hat{a}}_i^*) \\ &+ 2\sum_{i=1}^n (\hat{b}_i - b_{mi} - \hat{b}_i^*)^T Q_b^{-1}(\dot{\hat{b}}_i - \dot{\hat{b}}_i^*) + 2(\hat{c} - \hat{c}^*)^T Q_c^{-1}(\dot{\hat{c}} - \dot{\hat{c}}^*) + 2\varpi^T \hat{c}\end{aligned} \tag{12}$$

where the fact that $\dot{a}_m = \dot{b}_m = 0$ is used.

Letting

$$\dot{\hat{c}} - \dot{\hat{c}}^* = -Q_c \varpi \tag{13}$$

$$\dot{\hat{a}} - \dot{\hat{a}}^* = Q_a \varpi \dot{e}_i \tag{14}$$

$$\dot{\hat{b}} - \dot{\hat{b}}^* = Q_b \varpi e_i \tag{15}$$

and substituting them into equation (12), we have

$$\dot{V} = -x^T Q x + 2\hat{c}^{*T} \varpi - 2\sum_{i=1}^{n} a_i^{*T} \varpi \dot{e}_i - 2\sum_{i=1}^{n} b_i^{*T} \varpi e_i \tag{16}$$

Letting

$$\hat{c}^* = Q_c \varpi \tag{17}$$

$$\hat{a}_i^* = Q_a \varpi \dot{e}_i \tag{18}$$

$$\hat{b}_i^* = Q_b \varpi e_i \tag{19}$$

with Q_c, Q_a and Q_b being all positive definite, equation (16) becomes

$$\begin{aligned} \dot{V} &= -x^T Q x - 2\varpi^T Q_c \varpi - 2\sum_{i=1}^{n} (\varpi \dot{e}_i)^T Q_a \varpi \dot{e}_i - 2\sum_{i=1}^{n} (\varpi e_i)^T Q_b \varpi e_i \\ &\le -x^T Q x \le 0 \end{aligned} \tag{20}$$

From equation (20), we can conclude that the system (9) is stable and $x \to 0$ ($e \to e_m$ and $\dot{e} \to \dot{e}_m$) when $t \to \infty$. According to the reference model, we have $e \to 0$ and $\dot{e} \to 0$ when $t \to \infty$.

From equations (13) to (19), we have the following adaptation laws

$$\hat{c} = \hat{c}(0) - Q_c (\int_0^t \varpi(\tau) d\tau + \varpi) \tag{21}$$

$$\hat{A} = \hat{A}(0) + Q_a (\int_0^t \varpi(\tau) \dot{e}^T(\tau) + \varpi \dot{e}^T) \tag{22}$$

$$\hat{B} = \hat{B}(0) + Q_b (\int_0^t \varpi(\tau) e^T(\tau) + \varpi e^T) \tag{23}$$

Q. E. D

From equations (5) and (6), we have

$$f_d - f = M_m c - K_e e \tag{24}$$

As c is determined by f_d, r_e, r_d and M_m, it is bounded. Obviously $f_d - f$ is bounded as c is bounded and $e \to 0$.

Remarks:

1. The controller presented above can achieve the position tracking and the boundedness of the force tracking errors within the framework of impedance control approach. Compared with other controllers for position and force tracking, it has the advantages of impedance control such as the abilities to accommodate both unconstrained and constrained motion and the good robustness to disturbances [7].

5. SIMULATIONS

The simulation example is schematically shown in Figure 1. In the example, the end effector of the manipulator moves along a part of the constraint surface and exerts a force on it at the same time. The length, inertia and the mass of each link of the manipulator are $l_i = 0.3m$, $I_i = 0.3kgm^2$ and $m_i = 0.1kg$ respectively ($i = 1,2$). The mass center of each link is assumed to be in the middle of the link.

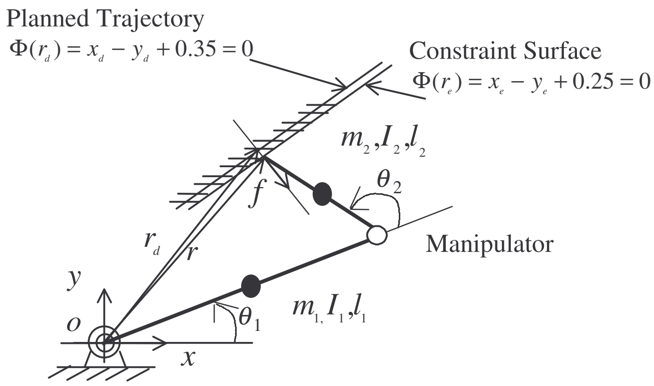

Figure 1 Simulation Example

In Figure 1, the world coordinate is denoted by oxy. The constraint surface is described by

$$\Phi(r_e) = x_e - y_e + 0.25 = 0 \tag{25}$$

and the planned trajectory of the end effector is

$$\Phi(r_d) = x_d - y_d + 0.35 = 0$$

Their trajectories in the time domain are represented by

$$x_d(t) = \tfrac{1}{10}\cos(2t), \quad y_d(t) = 0.35 - \tfrac{1}{10}\cos(2t)$$

$$x_e(t) = 0.05 - \tfrac{1}{10}\cos(2t), \; y_e(t) = 0.3 - \tfrac{1}{10}\cos(2t)$$

Assume that the rest position of the constraint surface is the same as that of the constraint surface (25). The planned force is set as $f_d = [f_{xd} \; f_{yd}]^T = [5\,5]^T$. The actual value of K_e is set as $150I^{2\times2}$.
The control parameters are chosen as follows: $Q_a = Q_b = Q_c = 1.5I^{2\times2}$, $A_m = 20I^{2\times2}$, $B_m = 400I^{2\times2}$, $P_1 = 1.25I^{2\times2}$ $P_2 = 6.56I^{2\times2}$, $\hat{A}(0) = 45I^{2\times2}$, $\hat{B}(0) = 30I^{2\times2}$, $\hat{c}(0) = [1\,1]^T$.

The position tracking and the force tracking performances are plotted in Figures 2 and 3 respectively. The control torques for the manipulators are given in Figure 4. It can be seen that under the proposed controller and the adaptation law, the positions converge to their desired values and the force errors are bounded. The control torques are in the reasonable ranges.

To examine the robustness of the controller, considering the external disturbances $\tilde{f}$ in the system dynamics such that

$$M_r(q)\ddot{r} + C_r(q,\dot{q})\dot{q} + G_r(q) = J^{-T}(q)\tau + f + \tilde{f}$$

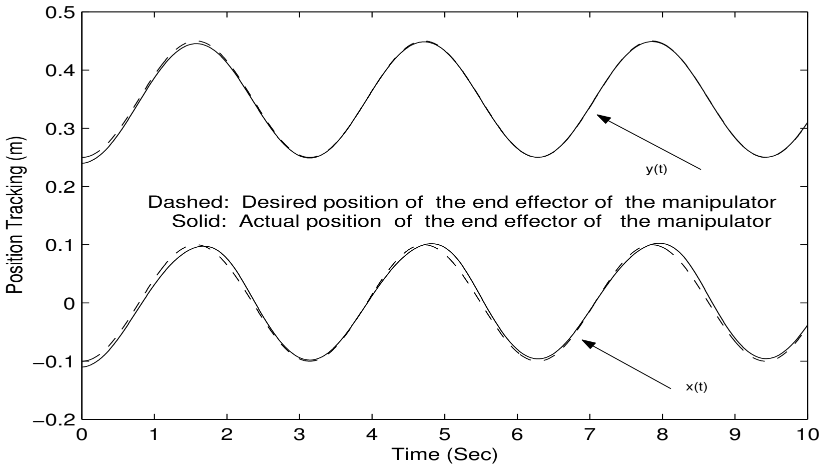

Figure 2 Position Tracking

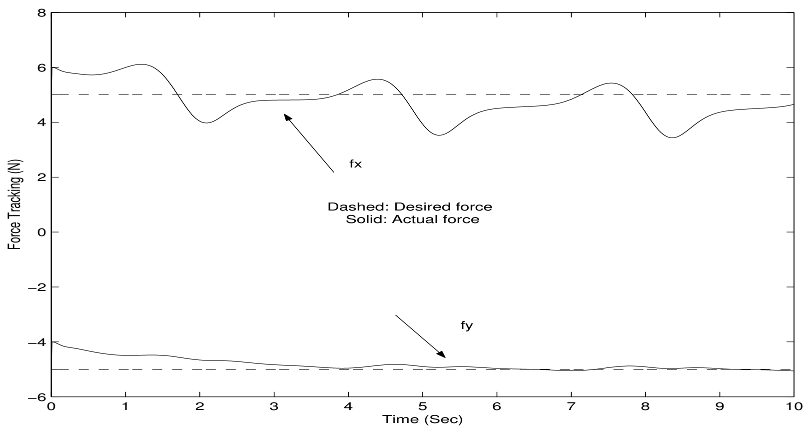

Figure 3 Force Tracking

Assume that the disturbances are bounded such that $\|\tilde{f}\| <= 0.25$ and all other conditions of the system remain the same. The simulation is done again and the results of position/force tracking are plotted in Figures 5 and 6 respectively. The control torques for the manipulators are given in Figure 7. Compared with the results for the cases without external disturbances, the performance of the controller is satisfactory. The position tracking is still maintained except for some variations on the tracking of $x(t)$. The force errors are kept bounded though the bounds become bigger. Control torques show bigger alternations, but they are in the reasonable ranges.

.

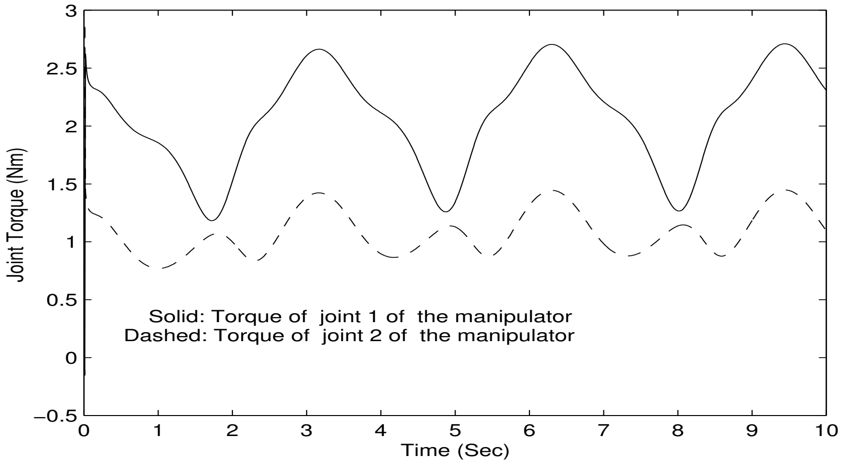

Figure 4 Joint Torques of the Robotic Arm

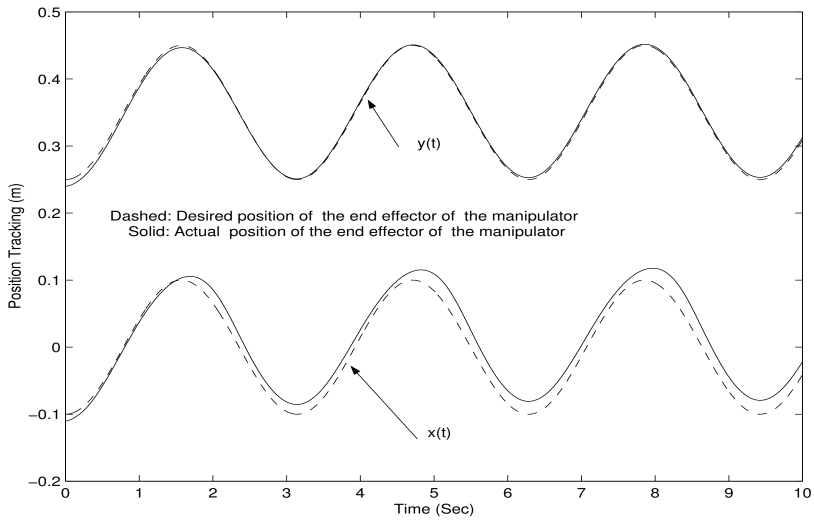

Figure 5 Position Tracking with External Disturbances

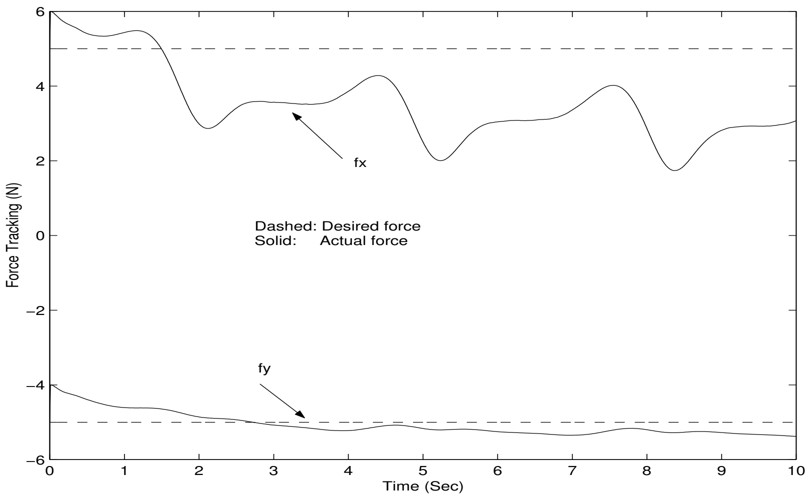

Figure 6 Force Tracking with External Disturbances

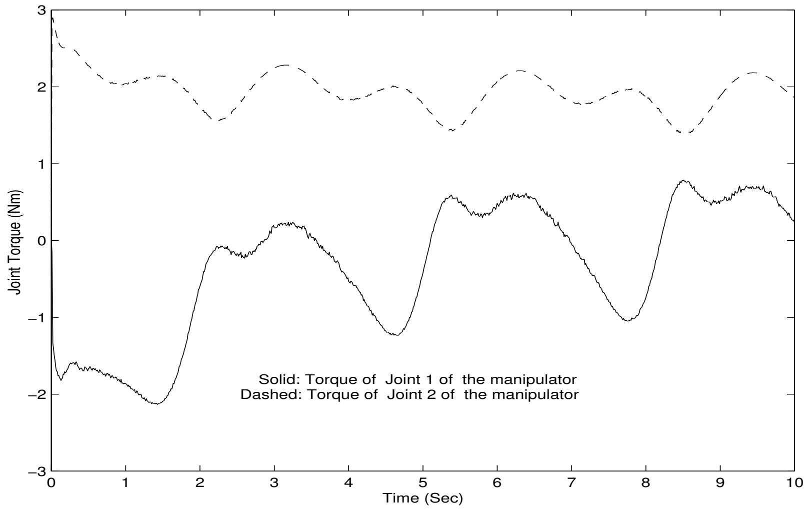

Figure 7 Joint Torques of the Robotic Arm with External Disturbances

6. CONCLUSIONS

In this paper, an adaptive impedance control scheme has been presented for the control of a constrained robot. The parameters of the constraint environment are unknown and the desired impedance was treated as time varying and was adapted with the robot's position tracking error. Under the proposed controller, the position of the robot converge to its desired trajectory and the constraint force error is bounded. Simulation results have verified the effectiveness of the control scheme.

REFERENCES

[1] N. Hogan, "Impedance control, an approach to manipulation: Part I—III ", *Journal of Dynamic Systems, Measurement, and Control,* Vol 107, pp. 1--24, 1985.

[2] R. Anderson and M. W. Spong, "Hybrid impedance control of robotic manipulators", *Proc. of the 1987 IEEE Conf. on Robotics and Auto*., pp. 1073-1080.

[3] C. Canudas de Wit, B. Siciliano and G. Bastin(Eds), *Theory of Robot Control,* Springer, 1996.

[4] W.-S. Lu and Q.-H. Meng, "Impedance control with adaptation for robotic manipulators", *IEEE Trans. on Robotics and Auto.*, Vol 7, No 3, pp. 408--415, 1991.

[5] Z. Lu, S. Kawamura and A. A. Goldenberg, "An approach to sliding-based impedance control", *IEEE Trans. on Robotics and Auto*., Vol 11, No 5, pp. 754--759, 1995.

[6] S. S. Ge, C. C. Hang, L. C. Woon and X. Q. Chen, "Impedance control of robot manipulators using adaptive neural networks," *International Journal of Intelligent Control and Systems*, Vol.2, No.3, pp.433-452, 1998.

[7] N. Hogan, "On the stability of manipulators performing contact task", *IEEE J. of Robot. Automat*., 4(6), pp. 677--686, 1988.

[8] H. Seraji and R. Colbaugh, "Force tracking in impedance control", *The Int. J of Robotics Research*, Vol 16, No 1, pp. 97--117, 1997.

[9] H.-P. Huang and S.-S. Chen, "Compliant motion control of robots by using variable impedance", *Int. J. Adv. Manuf. Technol,* Vol 7, pp. 322--332, 1992.

[10] H. K. Khalil, *Nonlinear Systems*, Prentice Hall, NJ, 1996.

[11] S.S. Ge, T.H. Lee and C.J. Harris, Adaptive Neural Network Control of Robotic Manipulators, World Scientific, London, 1998.

[12] S.S. Ge, L. Huang and T.H. Lee, "Model-based and neural-based adaptive control of two robotic arms manipulating an object with relative motion", Int. J. of Systems Science, vol. 32, no. 1, pp. 9-23, 2001.

Received: Mar 10th 2004

Accepted in final format: Nov 20th 2004

About the authors:

L. Huang is a lecturer in the School of Electrical and Electronic Engineering, Singapore Polytechnic, Singapore. He holds a Ph.D from National University of Singapore. He has been active in the development of robotic soccer systems, service robots and other autonomous mobile robots. The robotic soccer systems his team developed have won Gold medal, Silver medal and other awards in national and international competitions. Some robotic systems are commercialized and sold in Singapore, India, New Zealand and China. He has authored and co-authored about thirty international journal and conference papers and held one patent. His research interests are control of robotic manipulators and mobile robots and their real time applications. He can be reached by email at loulin@sp.edu.sg

S. S. Ge is an Associate Professor in the Department of Electrical & Computer Engineering, the National University of Singapore. He holds a Ph.D. degree of Imperial College (DIC) from Imperial College of Science, Technology and Medicine, University of London. He has authored and co-authored over 200 international journal and conference papers, two monographs and co-invented three patents. He has been serving as Associate Editors of various international journals including IEEE Transactions on Control Systems Technology and IEEE Transactions on Automatic Control. He was the recipient of National Technology Award, University Young Research Award, and Temasek Young Investigator Award, Singapore. His current research interests are control of nonlinear systems, hybrid systems, neural/fuzzy systems, robotics, sensor fusion, and real-time implementation. He can be reached by email at elegesz@nus.edu.sg

T. H. Lee is a Professor in the Department of Electrical and Computer Engineering at the National University of Singapore. He is also currently Head of the Drives, Power and Control Systems Group in this Department; and Vice-President and Director of the Office of Research at the University. He holds a Ph.D. degree from Yale University. He has co-authored three research monographs, and holds four patents. He currently holds Associate Editor appointments in Automatica; the IEEE Transactions in Systems, Man and Cybernetics; Control Engineering Practice; the International Journal of Systems Science and Mechatronics journal. Professor Lee was a recipient of the Cambridge University Charles Baker Prize in Engineering. His research interests are in the areas of adaptive systems, knowledge-based control, intelligent mechatronics and computational intelligence. He can be reached by email at eleleeth@nus.edu.sg