A SATISFICING GAME THEORETIC FRAMEWORK FOR RETRIEVING RELEVANT OBJECTS FROM A DATABASE

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ABSTRACT
In this paper we consider the problem of retrieving objects from a database that respond to a pursued goal (relevancy) by the extractors or decision makers to form a short list from which the final selection will be made because it is well known that human beings are good discriminators when facing few objects but perform very poorly in face of a great number of objects. These problems arise in domains such as finance and e-business (selecting a group of firms from a stock exchange database in which to invest, extracting desired goods to buy from a website, etc.), logistics (selecting a group of potential suppliers for a given product from a group of suppliers), administration (selecting a set of potential sites where facilities such as hospital, school, airport ... can be located). Most of the time the objects to be extracted are described by many attributes that can be measured, observed or supplied by experts. The purpose of this paper is to establish a method that permit to extract, from a database, a short list of objects relevant to the pursued goal using information about attributes and decision makers' preferences. We argue that, for a given goal, there are, almost always, attributes that work towards this goal and those that work against the goal; from this observation we propose to use the satisficing game theory approach that is based on the notion of "being good enough" as the underlying mathematical tool to establish the extraction method. The main idea of the method that will be established in this paper is to exploit the positive (working towards the pursued goal)/negative (working against the pursued goal) properties of attributes along with decision makers' preferences expressed by weighting these attributes to construct two measures known as *selectability* (related to positive attributes) and *rejectability* (related to negative attributes) in the framework of satisficing game theory. The objects arguable of "being good enough" are those for which the selectability measure exceeds the rejectability measure in some sense.

Keywords: Objects Retrieval, Database, Decision Making, Multi Attributes, Satisficing Game.

1. INTRODUCTION

Retrieving objects (cars, computers, houses, suppliers, software packages, ...) that must respond to a desired goal from a set of objects described by a certain number of attributes is a common and non obvious problem opened to humans individually as well as collectively in general. In a rich information world of nowadays, it is relatively easy to collect a huge number of such objects and their attributes from different sources (web, magazine, news papers, ...). These data are usually organized as tables called databases formally defined by a 4-tuples as given by equation (1)

$$DB = < U, A, D, \rho > \tag{1}$$

where:

- U is a finite non empty set of objects (a universe),

- A is a finite non empty set of attributes,
- $D = \bigcup_{a \in A} D_a$ with D_a being the *domain* of attribute a (set of all possible values that attribute a is allowed to take),
- $\rho : U \times A \to D$ is the *information or value function* such that $\rho(u,a) \in D_a$, $\forall\, u \in U$, $a \in A$,
- any couple (a, v), $a \in A$, $v \in D_a$ is called a descriptor in *DB*.

It is worth noticing that a database *DB* is just a finite data table where columns represent attributes that can be measured, observed or supplied by human experts, and rows represent objects and the entry in column a and row u is $\rho(u,a)$. The problem of retrieving objects from such a database that respond to a desired goal is then equivalent to a multi attributes/criteria decision analysis (MCDA) problem with possible (almost always ?) conflicting attributes. But it is well known that humans are good selectors (discriminators) when there are few objects (alternatives) and few attributes and perform very poorly in the case of a great number of alternatives and/or attributes. On the other hand a huge database will certainly contain redundant or irrelevant objects in the spirit of pursued goal. It is then necessary to find a procedure to automatically extract, from a huge database, a list of few objects that are relevant to pursued goal before final selection by decision makers. There are different methods for extracting useful objects from or reducing the dimension of a database such as rough set approximation approach (Pawlak, 1982), fuzzy set theory approach (Bouchon, 1995) and references therein and all approaches based on the soft computing techniques and artificial intelligence. Besides these approaches there are many approaches proper to multi attributes/criteria decision analysis problems that can be grouped into two main categories described below.

- Approaches based on *value function*: roughly speaking, these techniques consider a numerical function π defined on the universe U such that

$$\pi(u) \geq \pi(v) \Leftrightarrow u \succ v \tag{2}$$

where "$u \succ v$" stands for "u is at least as good, with regard to the pursued goal, as v or u dominates v" leading to a weak order on U. The decision analysis process then consists in building such a function based on the attributes values and decision makers' preferences; there are many techniques employed in the literature for constructing such a function where a number of them suppose a particular form for π such as expected utility or additive value function; see for instance (Bouyssou et. al., 2001), (Saaty, 1980), (Steuer, 1986), (Roy, 1993), (Vincke, 1989), (Winston, 1994) and references therein.

- Approaches allowing incomparability and/or intransitivity known in the literature as outranking methods such as the family of ELECTRE procedures and PROMETHEE techniques (see (Bouyssou et. al., 2001), (Roy, 1993), (Vincke, 1989)); these techniques necessitate in general iterative interaction between decision makers and the analyst.

In this paper we will adopt another approach, similar to value function approaches, based on satisficing game theory (Stirling, 2003) that can handle multi decision makers with possible conflicting views of attributes and that consider the sensitivity analysis process. For this purpose we suppose that it is possible to divide the considered attributes into two subsets A_g that contains positive attributes (attributes working toward the pursued goal) and A_c the subset of negative attributes, those attributes that can be interpreted as the cost. A typical database considered in this paper is then defined by equation (3)

$$DB = \langle U, A_g \times A_c, D, \rho \rangle \tag{3}$$

Notice that this assumption is not restrictive as this is the case for many selection problems; for instance to select an appropriate car to be bought we often make a list of positive attributes (driving comfort, speed, robustness, etc.) and a list of negative attributes (price, gas consumption, maintainability, etc.) and then try to balance them. Moreover, we consider that the value function ρ takes its values in a subset of real numbers that is $D_a \subseteq \Re$ for any attribute a; this can be seen as restrictive as it apparently excludes qualitative characterization of attributes but in many selection situations, qualitative description can be transformed in numerical values by establishing a certain order using experts' judgement for instance.

The remainder of this paper is organized as follows: in the next section the concepts of satisficing game theory that are relevant to our problem are presented; the third section (main contribution of this paper) consider the procedure of putting a multi decision makers/multi attributes retrieval problem into the framework of satisficing game theory; the section four considers an application to show the feasibility of the approach and a conclusion is presented in the fifth section.

2. SATISFICING GAME THEORY

Satisficing is a term that refers to a decision making strategy where options, objects or alternatives are selected which are "good enough" instead of being the best (Stirling, 2003). This way of selecting objects falls into the framework of praxeology or the study of theory of practical activity (the science of efficient action) derived from the epistemic logic (branch of philosophy that classifies propositions on the basis of knowledge and belief regarding their content; for a proposition to be admissible it must be both believable and informative) and developed by (Stirling, 2003). Earlier considerations of similar rationality for acting known as bounded rationality can be found in works such as the first edition of (Simon, 1997). Here decision makers instead of looking for the best object, look for a satisficing object. Let us consider a universe of options, alternatives or objects *U*; then for each object $u \in U$, a selectability measure $p_S(u)$ and a rejectability measure $p_R(u)$ are defined to measure the degree to which *u* works towards success in achieving the decision makers' goal and cost associated with this object respectively. This pair of measures called satisfiability functions are non negative and normalized to sum to one on the universe *U* (Stirling, 2003). The following definition then gives the set of objects arguable to be "good enough" because for these objects, the "benefit" expressed by the function $p_S(u)$ exceeds the cost expressed by the function $p_R(u)$ with regards to an index of caution *q*.

Definition 1. *The satisficing set* $\Sigma_q \subseteq U$ *is the set of objects defined by equation (4)*

$$\Sigma_q = \{u \in U : p_S(u) \geq q p_R(u)\}. \tag{4}$$

But for a satisficing object there can exist other satisficing objects that are better (having more selectability and at most the same rejectability or having less rejectability and at least the same selectability) than the previous one; it is obvious that in this case any rational decision maker will prefer the later objects. So the interesting set is that containing satisficing objects for which there is no better objects: this is called the *satisficing equilibrium* set E_q^S. To define this set, let us define first, for any object $u \in U$, the set *B(u)* of objects that are strictly better than *u*,

$$B(u) = B_S(u) \bigcup B_R(u) \tag{5}$$

where $B_S(u)$ and $B_R(u)$ are defined by equation (6)

$$\begin{aligned} B_S(u) &= \{v \in U : p_R(v) < p_R(u) \text{ and } p_S(v) \geq p_S(u)\}, \\ B_R(u) &= \{v \in U : p_R(v) \leq p_R(u) \text{ and } p_S(v) > p_S(u)\}. \end{aligned} \tag{6}$$

The equilibrium set *E* (objects for which there are no strictly better objects) is then given by equation (7)

$$E = \{u \in U : B(u) = \varnothing\} \tag{7}$$

and the satisficing equilibrium set E_q^S is defined by equation (8)

$$E_q^S = E \cap \Sigma_q. \tag{8}$$

One may wonder how to choose the index of caution *q*; the following sensitivity analyst with regard to this index may help one for this purpose. Small values of the index of caution *q* will lead to lot of objects being declared satisficing whereas large values of *q* will reduce the number of satisficing objects. A sensitivity analysis can be carried up to determine the value q_{min} below which all the objects of *U* will be declared satisficing and a value q_{max} above which there are no satisficing objects. For all objects of *U* to be declared satisficing the following inequality (9)

$$p_S(u) \geq q p_R(u), \forall\, u \in U \Leftrightarrow q \leq q_{\min} = \min_{u \in U} \left(\frac{p_S(u)}{p_R(u)} \right) \tag{9}$$

must be verified so that for such an index of caution q we have $\Sigma_q = U$. On the contrary, there is no satisficing objects, that is $\Sigma_q = \varnothing$ if and only if the following inequality (10)

$$p_S(u) < qp_R(u), \forall u \in U \Leftrightarrow q > q_{\max} = \max_{u \in U}\left(\frac{p_S(u)}{p_R(u)}\right) \tag{10}$$

is verified. Finally if the index of caution q verifies $q \in [q_{\min}, \quad q_{\max}]$ then $\Sigma_q \subseteq U$. We see then that one can manage to have a satisficing set with appropriate number of objects by incrementally increasing the index q from q_{min}.

In the next section we will establish a method that puts the problem of evaluating and selecting objects described by many attributes from a huge database as defined in the introduction section into satisficing game theory framework by establishing a systematic method to compute satisfiability functions $p_S(u)$ and $p_R(u)$ for each object u as well as sensitivity analysis.

3. SATISFICING RETRIEVAL PROCESS

3.1. Defining preliminary data

In the real world, evaluating and selecting objects processes are made by a certain number of decision makers; this is the group decision making or selection problem. For instance strategic decisions in an enterprise are taken by the executive board members that can comprise, general manager, marketing manager, production manager, financial manager etc.; political decisions such as choosing a place to settle a new facility (school, hospital, airport, waste management utilities, etc.), sponsoring projects, etc. are made most of the time by an elected council. In the case of evaluation and selection, we will talk about group selection. The fundamental characteristic of group evaluation and selection is the possible conflicting interests among decision makers in terms of importance to assign to each negative attribute as well as to each positive attribute. Our purpose in this paper is to derive a method that integrates different point of view of decision makers expressed through weights assigned to attributes similar to what was done in (Tchangani, 2006) for production units evaluation by different decision makers. We assume that d decision makers express their preference (concerns) with regards to negative attributes and positive attributes through following weights (α_{kj} and β_{kj}) defined on the same scale for each class of attributes; but the scale does not need to be the same for the negative attributes and positive attributes:

− α_{kj} (*k=1, 2, .., d*; $a_j \in A_g$) is the weight assigned by decision maker k to the positive attribute $a_j \in A_g$; the more selectable the attribute a_j is, in the view of this decision maker, the more important is α_{kj};

− β_{kj} (*k=1, 2, .., d*; $a_j \in A_c$) is the weight assigned by decision maker k to negative attribute $a_j \in A_c$; the more rejectable the attribute a_j is, in the view of this decision maker, the more important is β_{kj}.

We do think that it is easier to ask to decision makers to compare attributes in order to express their preferences rather than to compare objects; these weights can be determined by using pairwise comparison method such as analytic hierarchy processes, see for instance (Saaty, 1980) as it has been shown by the author in (Tchangani, 2006a). These weights are then aggregated to define selectability weights ω_j^S and rejectability weights ω_j^R by taking the mean value over decision makers' preference as given by equation (11)

$$\omega_j^S = \frac{\sum_{k=1}^{d} \alpha_{kj}}{\sum_{a_j \in A_g} \sum_{k=1}^{d} \alpha_{kj}} \quad and \quad \omega_j^R = \frac{\sum_{k=1}^{d} \beta_{kj}}{\sum_{a_j \in A_c} \sum_{k=1}^{d} \beta_{kj}} \tag{11}$$

The weights ω_j^S and ω_j^R measure the aggregate strength that decision makers accord to positive attribute $a_j \in A_g$ and negative attribute $a_j \in A_c$ respectively with regard to other items of the same category. Let us define ω^S and ω^R as row vectors given by equation (12)

$$\omega^S = \left[\omega_1^S \quad \omega_2^S \quad \ldots \quad \omega_{|A_g|}^S\right] \quad and \quad \omega^R = \left[\omega_1^R \quad \omega_2^R \quad \ldots \quad \omega_{|A_c|}^R\right] \tag{12}$$

and functions $g_S(u)$ and $g_R(u)$ for each object $u \in U$ that work toward globally selecting u or globally rejecting u respectively as follows, equation (13)

$$g_S(u) = \omega^S g_u \quad and \quad g_R(u) = \omega^R c_u \tag{13}$$

where g_u and c_u defined by equation (14)

$$g_u = \left[g_u(1) \quad g_u(2) \quad \ldots \quad g_u(|A_g|)\right]^T \quad and \quad c_u = \left[c_u(1) \quad c_u(2) \quad \ldots \quad c_u(|A_c|)\right]^T \tag{14}$$

with

$$\begin{aligned} g_u(i) &= \frac{\rho(u,a_i) - \min_{v\in U} \rho(v,a_i)}{\max_{v\in U} \rho(v,a_i) - \min_{v\in U} \rho(v,a_i)}, \forall\, a_i \in A_g \\ c_u(i) &= \frac{\rho(u,a_i) - \min_{v\in U} \rho(v,a_i)}{\max_{v\in U} \rho(v,a_i) - \min_{v\in U} \rho(v,a_i)}, \forall\, a_i \in A_c \end{aligned} \tag{15}$$

are normalized column vectors of positive attributes and negative attributes values of object u respectively; x^T stands for transpose of vector x and $|M|$ is the cardinal (number of elements in M) of the set M. A normalization process is necessary before weighting because attributes do not necessarily have the same units (money, memory capacity, human resources, surface, machines, etc.). The following definition then gives important data through which retrieval and sensitivity analysis can be carried up.

Definition 2. *The satisfiability functions p_S and p_R of selection problem are defined by equation (16)*

$$p_S(u) = \frac{g_S(u)}{\sum_{x\in U} g_S(x)} \quad and \quad p_R(u) = \frac{g_R(u)}{\sum_{x\in U} g_R(x)} \tag{16}$$

the set of selectable objects Σ_q is defined by equation (17)

$$\Sigma_q = \{u \in U : p_S(u) \geq q p_R(u)\} \tag{17}$$

and the satisficing equilibrium objects set (relevant objects) E_q^S by equation (18)

$$E_q^S = E \cap \Sigma_q \quad with \quad E = \{u \in U : B(u) = \varnothing\} \tag{18}$$

where B(u) is defined by equation (5).

An important question that can be raised at this stage is that of coherence of this method: that is, if there is an object which negative attributes values are more important and which positive attributes values are less important than the corresponding values for another object, is there a chance that the former object be declared as a satisficing equilibrium object ? Let us consider the following definition that formalizes this idea.

Definition 3. *An object $u \in U$ dominates an object $v \in U$, noted $u \succ v$, if and only if the following inequalities (19)*

$$\rho(u,a_i) \geq \rho(v,a_i), \forall\, a_i \in A_g \quad and \quad \rho(u,a_j) \leq \rho(v,a_j), \forall\, a_j \in A_c \tag{19}$$

hold with at least one strict inequality.

The following theorem establishes the coherency of the method: a dominated object cannot be declared as a satisficing equilibrium object.

Theorem. *Let u and v belong to U then* $u \succ v \Rightarrow u \in B(v)$ *and so* $v \notin E$.

Proof. $u \succ v \Rightarrow g_u(i) \geq g_v(i) \geq 0 \ \forall \ a_i \in A_g$ *and* $0 \leq c_u(j) \leq c_v(j) \ \forall \ a_j \in A_c$ *with at least one strict inequality and as* $\omega_l^S \geq 0,\ \omega_l^R \geq 0$, *we have* $g_S(u) \geq g_S(v)$ *and* $g_R(u) \leq g_R(v)$ *and finally* $p_S(u) \geq p_S(v)$ *and* $p_R(u) \leq p_R(v)$ *with at least one strict inequality so* $u \in B(v)$ *that is* $B(v) \neq \varnothing$ *and v is not an equilibrium.*

The evaluation and retrieval process as well as sensitivity analysis can be done now.

3.2. Retrieval and sensitivity analysis

Necessary information for evaluation and selection process are summarized in the sets Σ_q, E and E_q^S as well as $B(u)$.

- Objects of the set E_q^S are those one can qualify as "good enough" and the core of selected objects: their selectability exceeds their rejectability and there are no objects that are better than them.
- Σ_q is the set of objects that are satisficing (their selectability exceeds their rejectability) but are not necessarily non dominated by other objects. If an object $u \notin \Sigma_q$, one can do a sort of sensitivity analysis to determine the way to render it relevant by computing the amount by which its positive attributes values must be increased and the amount by which its negative attributes values must be reduced in order to be satisficing if other objects attributes values remain unchanged. To do so, one can compute sensitivity parameters $\delta_u^i \geq 0, i = 1, 2, .., |A_g|$ and $\gamma_u^i \geq 0, i = 1, 2, .., |A_c|$, such that, if one replaces $g_u(i)$ and $c_u(i)$ by $g_u(i) + \delta_u^i$ and $c_u(i) - \gamma_u^i$ respectively when satisfying the conditions of equation (20)

$$0 < g_u(i) + \delta_u^i \leq 1 \quad and \quad 0 < c_u(i) - \gamma_u^i \leq 1 \tag{20}$$

then

$$p_S(u) \geq q p_R(u). \tag{21}$$

One can find these parameters by solving the following nonlinear program (22),

$$\min_{\delta_u, \gamma_u} 0 \quad s.t. \quad \begin{cases} C_g(\delta_u) \geq q C_c(\delta_u) \\ \varepsilon_g \leq g_u + \delta_u \leq 1,\ \delta_u \geq 0 \\ \varepsilon_c \leq c_u - \gamma_u \leq 1,\ \gamma_u \geq 0 \end{cases} \tag{22}$$

where

$$\delta_u = \begin{bmatrix} \delta_u^1 & \delta_u^2 & \ldots & \delta_u^{|A_g|} \end{bmatrix}^T \quad and \quad \gamma_u = \begin{bmatrix} \gamma_u^1 & \gamma_u^2 & \ldots & \gamma_u^{|A_c|} \end{bmatrix}^T \tag{23}$$

1 (respect. 0) in the second part of the equation (22) is a column vector with appropriate dimension and all entries equal to 1 (respect. 0); ε_g and ε_c are vectors with appropriate dimensions expressing lower bounds on positive attributes and negative attributes respectively; s.t. stands for "subjected to"; and finally the nonlinear functions $C_g(\delta_u)$ and $C_c(\gamma_u)$ are given by the following equation (24)

$$C_g(\delta_u) = \frac{\omega^S(g_u + \delta_u)}{\sum_{v \in U, v \neq u} \omega^S g_v + \omega^S(g_u + \delta_u)} \quad and \quad C_c(\gamma_u) = \frac{\omega^R(c_u - \gamma_u)}{\sum_{v \in U, v \neq u} \omega^R c_v + \omega^R(c_u - \gamma_u)}. \tag{24}$$

Notice that, this program is in very general form and other constraints can be added to take into account practical requirements such as uniform distribution of effort for a class of attributes for instance or on the contrary concentrating the effort on some particular attributes. This analysis is well suited for objects of the set $E - E_q^S$ (objects for which there is no other objects that are better but are not satisficing); $\frac{\delta_u(i)}{g_u(i)}$ and $\frac{\gamma_u(i)}{c_u(i)}$ are amount by which positive attributes values (normalized) of object u must be increased and the amount by which its negative attributes values (normalized) must be reduced respectively when performance of all other objects remain unchanged in order to be satisficing.

- Sets *B(u)* may be with great importance to decision makers and/or suppliers of objects because they can use them to identify weak points of irrelevant objects and possible causes of this weakness. For instance if $u^* \in B(u)$, by comparing the environments from which these objects come from, one can identify why (causes) object u^* is performing better than object u mainly for those objects of the set $\Sigma_q - E_q^S$. A procedure similar to that presented in the previous point can be used to look for how u can be rendered as good as u^*, that is, determine parameters $\delta_u^{u^*}$ and $\gamma_u^{u^*}$ (defined as δ_u and γ_u respectively in previous point) so that

$$p_S(u) = \frac{\omega^S\left(g_u + \delta_u^{u^*}\right)}{\sum\limits_{v\in U, v\neq u} \omega^S g_v + \omega^S\left(g_u + \delta_u^{u^*}\right)} = p_S(u^*)$$
$$p_R(u) == \frac{\omega^R\left(c_u - \gamma_u^{u^*}\right)}{\sum\limits_{v\in U, v\neq u} \omega^R c_v + \omega^R\left(c_u - \gamma_u^{u^*}\right)} = p_R(u^*) \quad (25)$$

which can be done by solving the following linear programming problem (26)

$$\min_{\delta_u^{u^*}, \gamma_u^{u^*}} 0 \quad s.t. \quad \begin{cases} \omega^S \delta_u^{u^*} = \dfrac{p_S(u^*)\left(\sum_{v\in U} \omega^S g_v\right) - \omega^S g_u}{1 - p_S(u^*)} \\ \omega^R \gamma_u^{u^*} = -\dfrac{p_R(u^*)\left(\sum_{v\in U} \omega^R c_v\right) - \omega^R c_u}{1 - p_R(u^*)} \\ \varepsilon_o \le g_u + \delta_u^{u^*} \le 1, \ \delta_u^{u^*} \ge 0 \\ \varepsilon_i \le c_u - \gamma_u^{u^*} \le 1, \ \gamma_u^{u^*} \ge 0 \end{cases} \quad (26)$$

- The set $U - \Sigma_q \cup E$ contains completely irrelevant objects; they are not satisficing objects nor equilibrium objects.

Remark. *Notice that optimization problems (22) and (26) are mathematically ill-posed problems (many solutions); by using other criteria and/or constraints, for instance uniform distribution of weights* δ_u *and* γ_u *or* $\delta_u^{u^*}$ *and* $\gamma_u^{u^*}$*, lower and upper bounds etc., one can ensure a well posedness. When an object's attributes values are changed, the configuration of the problem may change.*

3.3 Comments

The index of caution q will permit decision makers to modulate their aspiration; if few objects are declared satisficing for an index q, decision makers may diminish this index to increase the number of satisficing objects; on the contrary if too many objects are declared satisficing, this index may be increased to reduce their number. This procedure can be programmed to incrementally vary the index of caution from q_{min} until the appropriate number of the objects to be

included in the short list is obtained. How this number is determined will depend on the problem at hand and the preferences of decision makers; this approach will be used essentially in the preparatory phase of a selection problem where a short list of objects are to be obtained before a discussion by decision makers for final choice. The sensitivity analysis gives possibility for negotiation: let us suppose for instance that, the universe of discourse U of the database in hand is constituted by manufactured objects or services to be purchased by decision makers, each object of the universe coming from a given supplier or provider. A supplier or a provider whose object is declared not satisficing may want to ameliorate its offer by changing one or more of its object's attributes; the sensitivity analysis in terms of parameters δ_u and γ_u allows decision makers to quickly judge the opportunity of the new offer. On the other hand, a provider or supplier who knows that his/her object u is dominated by an object $u*$ of another provider or supplier (that is $u* \in B(u)$) can try to do better by computing parameters δ_u^{u*} and γ_u^{u*}. Another possibility offered by the sensitivity analysis is that regarding the variation of weights supplied by decision makers; one may analyze how the selection problem structure changes if one or more decision makers change their weights. Furthermore all the processes (weights supply process, construction of the database and possibly negotiation process) presented can be carried up in distributed manner where decision makers on one hand and suppliers on the other hand can be remotely distributed and communicate using information and communication technology devices. The database can be constructed by searching data (objects on one hand and their attributes values on the other hand) from remote distributed computer servers.
The approach presented so far has the following positive points:

- it is easy to understand and to integrate in a decision support system;
- preferences are expressed locally (for each object) by decision makers rather than globally as it is often done in multi criteria decision making literature;
- one knows for a dominated object, objects that perform better and so one can analyze the reasons of its weakness;
- the sensitivity analysis gives valuable information to decision makers;
- it does not necessitate important computation power;

some of its negative points could be the following:

- it is required to normalize original data;
- satisfiability functions do not express a meaningful parameters for the object.

In the next section an application of this method to a financial problem that consists in retrieving relevant firms from a stock exchange database for investment purpose is considered.

4. APPLICATION TO PORTFOLIO MANAGEMENT

Modern theory of portfolio management has been initiated in 1952 by Markowitz, (Markowitz, 1952) when he proposed his mean-variance model for selection purpose. According to this theory, any portfolio manager should seek the optimization of two conflicting criteria: maximize the mean return and minimize the risk measured by the variance of this return. Thus, managing a portfolio is a multi attributes decision problem (Hurson and Zopounidis, 1997) and actually an efficient management of portfolio must consider more than two conflicting criteria because each firm is determined by different performance indices that must be optimized by portfolio manager when selecting firms for investment. Investment decision is made in two stages: at the first stage the portfolio manger selects a set of firms from a stock exchange database for instance and in a second stage solves an affectation problem that is which proportion of his fund will be invested in each selected firm. Here we are interested by the first stage process. Two main performance indices are used in practice: financial performance indices, namely:

- current ratio (**CR**), a criterion related to cash that must be maximized;
- return on equity (**ROE**), capital profitability criterion that must be maximized;
- cash flow over liability ratio (**CFLR**), a creditworthiness criterion that must be maximized;

and stock exchange performance indices given by:

- earnings per share (**EPS**) that must be maximized;
- monthly mean return (**MMR**) that must be maximized;

- *β-1* (**Beta-1**) a technical coefficient which absolute value must be minimized;
- price earning ratio (**PER**) that must be minimized.

Let us consider data of 31 firms of a certain stock exchange given on Figure 1 (except the two last columns) where we distinguish positive attributes corresponding to criteria to be maximized and negative attributes corresponding to criteria to be minimized. These data are extracted from (Hurson and Zopounidis, 1997) where a multi criteria decision making process were used to rank these firms; notice that the intention here is not to compare our approach to that used in (Hurson and Zopounidis, 1997) but just to simulate a possible used of our approach with a sensitivity analysis.

Firm	***Positive attributes***					***Negative attributes***		***Satisfiability functions***	
	CR	**ROE**	**CFLR**	**EPS**	**MMR**	**Beta - 1**	**PER**	p_S	p_R
X01	1.2300	0.1470	4.9100	9538	0.0950	0.3000	0.8097	0.0672	0.0189
X02	1.3600	0.0980	0.7400	518	0.0110	0.1060	-8.0645	0.0288	0.0094
X03	0.8500	0.1410	0.1900	600	0.0190	0.0130	32.2581	0.0260	0.0251
X04	0.9700	0.1180	0.6900	328	0.0090	0.1580	22.7273	0.0254	0.0248
X05	1.6300	0.2300	0.2600	10762	0.0240	0.0660	11.6279	0.0535	0.0172
X06	0.7200	0.2410	0.6400	105	0.0100	0.1010	52.6316	0.0258	0.0368
X07	0.8900	0.1630	0.5200	68	-0.0020	0.0700	20.8333	0.0232	0.0215
X08	1.1000	0.2120	0.8800	1312	0.0130	0.0310	7.4627	0.0314	0.0143
X09	1.3100	0.2020	1.7200	2335	0.0120	0.0970	9.4340	0.0363	0.0171
X10	1.5700	0.1370	0.5800	1018	0.0180	0.0790	13.5135	0.0331	0.0184
X11	0.8200	0.1710	0.8800	639	0.0030	0.1560	19.6078	0.0254	0.0233
X12	1.2800	0.1770	0.3100	86	0.1100	5.7400	2.2883	0.0438	0.1726
X13	1.5800	0.2160	0.3200	217	-0.0010	0.1310	30.3030	0.0299	0.0275
X14	1.4100	0.1860	0.2400	168	-0.0010	0.2050	-22.2222	0.0276	0.0057
X15	1.0700	0.1810	0.1900	2651	0.0070	0.0170	58.8235	0.0304	0.0372
X16	1.1000	0.1770	1.0100	859	0.0170	0.1400	11.1111	0.0307	0.0190
X17	2.6000	0.1640	0.5100	25	0.0050	0.0020	13.1579	0.0379	0.0161
X18	1.0600	0.1140	0.3400	212	-0.0010	0.1760	19.2308	0.0235	0.0237
X19	1.4300	0.2990	1.6600	294	0.0100	0.0900	19.2308	0.0353	0.0213
X20	1.0400	0.0640	0.7100	168	0.0010	0.0840	18.1818	0.0233	0.0207
X21	1.8700	0.1040	0.3100	235	0.0020	0.0590	16.3934	0.0302	0.0191
X22	0.6800	-0.5700	0.9600	-88	-0.0150	0.2880	2.3866	0.0036	0.0192
X23	0.6400	0.1500	0.2300	316	0.0110	0.0640	-0.5851	0.0228	0.0116
X24	2.4800	0.1500	9.4100	371	0.0050	0.3500	13.6986	0.0562	0.0261
X25	1.9100	0.0660	4.8700	127	-0.0060	0.4170	55.5556	0.0379	0.0470
X26	0.4300	0.1120	0.8200	176	0.0050	0.6560	333.3333	0.0203	0.1799
X27	0.4400	0.0750	1.3600	139	0.0080	0.8080	25.0000	0.0211	0.0441
X28	0.7400	0.0250	2.9900	125	0.0020	0.1920	13.3333	0.0249	0.0215
X29	2.8800	0.1720	3.6700	1485	0.0040	0.0160	10.1010	0.0495	0.0151
X30	2.3100	0.1630	0.6200	3155	0.0420	0.2070	20.8333	0.0473	0.0253
X31	0.8500	0.1520	1.3100	687	0.0100	0.1720	12.3457	0.0273	0.0205

Figure 1: Financial and stock exchange performance indices of 31 firms of a certain stock exchange.

In (Hurson and Zopounidis, 1997) two methods were used for the first stage (selection of firms where to invest) purpose: MINORA that uses interactive UTA algorithms (see (Jacquet-Lagrèze and Sisko, 1982) for preference breakup and an outranking method ELECTRE TRI (see (Yu, 1992) and finally ADELAIS (see (Siskos and Despotis, 1989), an interactive method for multi objectives linear programming is used for second stage process.

As there is not a portfolio management specialist who could advice us about the importance of attributes, we consider that all attributes have equal importance. From this assumption, satisfiability functions p_S and p_R obtained are given by the two last columns of Figure 1 respectively and we find that $q_{min} = 0.1130$ and $q_{max} = 4.8389$. The equilibrium firms set (firms that are not dominated) is given by equation (27)

$$E = \{X_{01}, X_{02}, X_{05}, X_{08}, X_{14}, X_{29}\}. \qquad (27)$$

For an index of caution $q = 1$, only *8* firms are not satisficing that are given by equation (28)

$$U - \Sigma_1 = \{X_{06}, X_{12}, X_{15}, X_{18}, X_{22}, X_{25}, X_{26}, X_{27}\}; \qquad (28)$$

we can see that too many firms are satisficing at this index of caution. Let us suppose that the portfolio manager consider an index of caution of $q = 3$ by incremental procedure then

$$\Sigma_3 = \{X_{01}, X_{02}, X_{05}, X_{14}, X_{29}\} \quad and \quad E_3^S = \Sigma_3 . \qquad (29)$$

A sensitivity analysis can be done to see how to render the firm given by equation (30)

$$E - E_3^S = \{X_{08}\} \tag{30}$$

satisficing. By solving the problem (22) for the firm X_{08} we obtain

$$\delta_{X_{08}} = \begin{bmatrix} 0.0009 \\ 0.0009 \\ 0.0009 \\ 0.0009 \\ 0.0009 \end{bmatrix} \quad and \quad \frac{\delta_{X_{08}}}{g_{X_{08}}} = \begin{bmatrix} 0.0031 \\ 0.0009 \\ 0.0114 \\ 0.0066 \\ 0.0038 \end{bmatrix}$$
$$\gamma_{X_{08}} = \begin{bmatrix} 0.0050 \\ 0.0190 \end{bmatrix} \quad and \quad \frac{\gamma_{X_{08}}}{c_{X_{08}}} = \begin{bmatrix} 0.9802 \\ 0.2276 \end{bmatrix} \tag{31}$$

where $\frac{\delta_{X_{08}}}{g_{X_{08}}}$ and $\frac{\gamma_{X_{08}}}{c_{X_{08}}}$ are taken componentwise. One can see that the reason of X_{08} being no satisficing is mainly due to its negative attributes; for instance normalized price earning ratio (PER) should be reduced by *22.76 %* and *β-1* reduced by *98.02 %* in order for it to become satisficing when other firms performance remain unchanged.

In (Hurson and Zopounidis, 1997) it was reported that, the ranking of the six first firms by a portfolio manger was the following, equation (32)

$$\{X_{01}, X_{24}, X_{05}, X_{30}, X_{29}, X_{09}\}. \tag{32}$$

Notice that for $q = 1$, all these firms are satisficing but $\{X_{24},\ X_{30}, X_{09}\}$ are not equilibria and we have results of equation (33)

$$\begin{aligned} B(X_{24}) &= \{X_{01}\}, \\ B(X_{30}) &= \{X_{01}, X_{05}, X_{29}\}, \\ B(X_{24}) &= \{X_{17}, X_{29}\}. \end{aligned} \tag{33}$$

Solving a problem similar to (26) for X_{24} we obtain the following results (34):

$$\delta_{X_{24}}^{X_{01}} = \begin{bmatrix} 0.0294 \\ 0.0306 \\ 0.0000 \\ 0.2984 \\ 0.2439 \end{bmatrix} \quad and \quad \frac{\delta_{X_{24}}^{X_{01}}}{g_{X_{24}}} = \begin{bmatrix} 0.0351 \\ 0.0370 \\ 0.0000 \\ 7.0529 \\ 1.5242 \end{bmatrix}$$
$$\gamma_{X_{24}}^{X_{01}} = \begin{bmatrix} 0.0169 \\ 0.0286 \end{bmatrix} \quad and \quad \frac{\gamma_{X_{24}}^{X_{01}}}{c_{X_{24}}} = \begin{bmatrix} 0.2783 \\ 0.2835 \end{bmatrix} \tag{34}$$

that show that, in order for X_{24} not to be dominated by X_{01}, its positive attributes **EPS** and **MMR** among other, must be augmented by *705.29%* and *152.42%* respectively. This result can be validated when looking at original data of Figure 1 where X_{01} positive attributes **EPS** and **MMR** values exceed largely the corresponding values of X_{24}. This study can be carried up for other dominated firms.

This method is suited for portfolio management as it is able to classify firms in *3* categories: those firms that definitely can not be included in the portfolio (objects of $U - \Sigma_q \cup E$), firms that can be included in the portfolio (objects of $\Sigma_q \cap E$) and other firms for which there is some uncertainty as it is the case in practice (Hurson and Zopounidis, 1997).

3. CONCLUSION

We have proposed in this paper a satisficing game theoretic framework for retrieving relevant objects described by several attributes from a large database to form a short list to be presented to decision makers for final selection that will respond to some pursued goal. The method allows many decision makers with possible conflicting points of view regarding the importance to assign to each attribute to express their concerns and preferences. The main difference of this approach to other approaches of the multi criteria decision analysis literature applying satisficing game theory,is that they rely on the process of dividing attributes into positive attributes and negative attributes that permitted a systematic method to compute satisfiability functions (selectability and rejectability functions), from the information function while deriving the preferences of decision makers expressed by weighting attributes. This method allows decision makers to modulate the number of extracted objects by varying the index of caution from a minimum value below which all objects are satisficing to a maximum value above which there are no satisficing objects. The sensitivity analysis provided may be very valuable in practice to the decision makers as well as to the objects providers as it can be used for negotiation purpose. An application considered in the domain of portfolio management shows the feasibility of the established method. This approach is suited for the preliminary stage of a selection problem where a short list of objects must be selected from a huge database that will be presented to decision makers for final decision process and it can easily be integrated in distributed decision support systems that can be used as an aid for decision making and negotiation; this problem will be considered in future works.

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Received: May 12th 2006

Accepted in final format: January 29th 2007 after one revision.

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